Example: With $f(x,y) = \frac{7}{8}(y^2+y+x)^4$

compute $f_{xyxyx}(2,0)$.

Solution 1:

$$f_x(x,y) = \frac{7}{2}(y^2+y+x)^3$$

$$f_{xy}(x,y) = \frac{21}{2}(y^2+y+x)^2(2y+1)$$

$$f_{xyx}(x,y) = 21(y^2+y+x)(2y+1)$$

$$f_{xyxy}(x,y) = 21(2y+1)^2 + 42(y^2+y+x)$$

$$f_{xyxyx}(x,y) = 42$$

$$f_{xyxyx}(2,0) = 42.$$

Solution 2: $f_{xyxyx}(2,0) = f_{xxxyy}(2,0)$

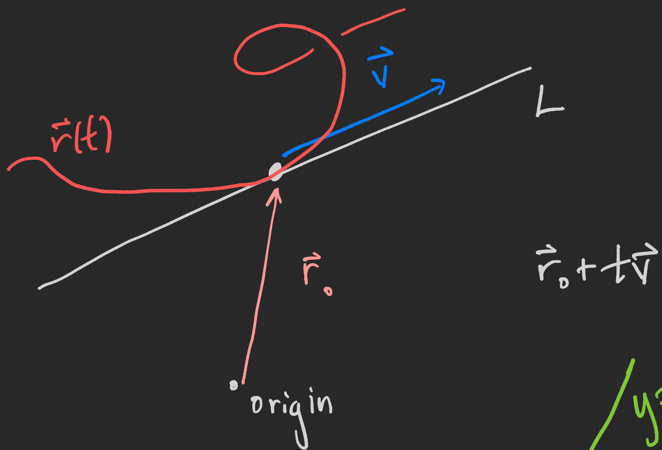
$$f_{xx}(x,y) = \frac{21}{2}(y^2+y+x)^2$$

$$f_{xxx}(x,y) = 21(y^2+y+x)$$

$$f_{xxxy}(x,y) = 42y + 21$$

$$f_{xxxyy}(x,y) = 42 \qquad f_{xxxyy}(2,0) = 42.$$

(So we've verified in this particular example that Clairaut's Thm holds.)



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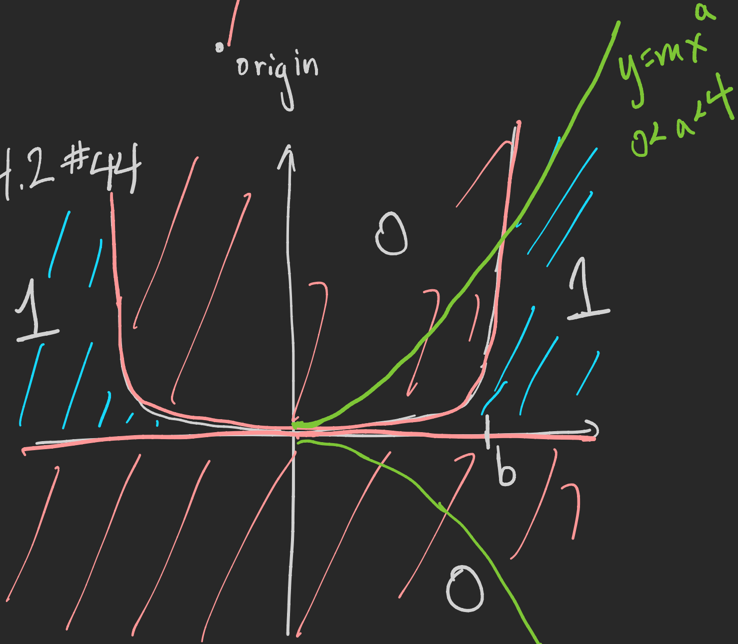

If $m > 0$:

Show there exists $b > 0$ s.t.

$$mx^a \geq x^4 \quad \text{for all } 0 < x < b$$

(so $f(x,y) = 0$)

If $m \leq 0$:

Then $mx^a \leq 0$. (so $f(x,y) = 0$)

In either case, the limit along towards $(0,0)$ is 0.